Mark Scheme (Results)

November 2021

Pearson Edexcel GCE

In AS Further Mathematics (8FM0)

Paper 01 Core Pure Mathematics

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 80.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1(a)(i)	Rotation	B1	1.1b
	90 degrees anticlockwise about the origin	B1	1.1b
(ii)	Stretch	B1	1.1b
	Scale factor 3 parallel to the y-axis	B1	1.1b
		(4)	
(b)	$\mathbf{QP} = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 3 & 0 \end{pmatrix}$	B1	1.1b
		(1)	
(c)(i)	$\lvert\mathbf{R}\rvert = 3$	B1ft	1.1b
(ii)	The **area** scale factor of the transformation	B1	2.4
		(2)	
			(7 marks)

Notes

(a)(i)
B1: Identifies the transformation as a rotation
B1: Correct angle (allow equivalents in degrees or radians), direction and centre the origin
(ii)
B1: Identifies the transformation as a stretch
B1: Correct scale factor and parallel to/in/along the y-axis/y direction
(b)
B1: Correct matrix
(c)(i)
B1ft: Correct value for the determinant (follow through their **R**)
(ii)
B1: Correct explanation, must include **area**
Note: scale factor of the transformation is B0

Question	Scheme	Marks	AOs
2	$w=3x-2 \Rightarrow x=\dfrac{w+2}{3}$	B1	3.1a
	$9\left(\dfrac{w+2}{3}\right)^3-5\left(\dfrac{w+2}{3}\right)^2+4\left(\dfrac{w+2}{3}\right)+7=0$	M1	3.1a
	$\dfrac{1}{3}\left(w^3+6w^2+12w+8\right)-\dfrac{5}{9}\left(w^2+4w+4\right)+\dfrac{4}{3}(w+2)+7=0$		
	$3w^3+13w^2+28w+91=0$	dM1 A1 A1	1.1b 1.1b 1.1b
		(5)	
	Alternative:		
	$\alpha+\beta+\gamma=\dfrac{5}{9}, \alpha\beta+\beta\gamma+\alpha\gamma=\dfrac{4}{9}, \alpha\beta\gamma=-\dfrac{7}{9}$	B1	3.1a
	New sum = $3(\alpha+\beta+\gamma)-6=-\dfrac{13}{3}$	M1	3.1a
	New pair sum = $9(\alpha\beta+\beta\gamma+\gamma\alpha)-12(\alpha+\beta+\gamma)+12=\dfrac{28}{3}$		
	New product = $27\alpha\beta\gamma-18(\alpha\beta+\beta\gamma+\gamma\alpha)+12(\alpha+\beta+\gamma)-8=-\dfrac{91}{3}$		
	$w^3-\left(-\dfrac{13}{3}\right)w^2+\dfrac{28}{3}w-\left(-\dfrac{91}{3}\right)=0$	dM1	1.1b
	$3w^3+13w^2+28w+91=0$	A1 A1	1.1b 1.1b
		(5)	
			(5 marks)

Notes

B1: Selects the method of making a connection between x and w by writing $x=\dfrac{w+2}{3}$

Condone the use of a different letter than w

M1: Applies the process of substituting $x=\dfrac{w+2}{3}$ into $9x^3-5x^2+4x+7=0$

dM1: Depends on the previous M mark. Manipulates their equation into the form $aw^3+bw^2+cw+d\,(=0)$. Condone the use of a different letter then w consistent with B1 mark.

A1: At least two of a, b, c, d correct
A1: Fully correct equation, must be in terms of w
Alternative:
B1: Selects the method of giving three correct equations containing α, β and γ
M1: Applies the process of finding the new sum, new pair sum, new product
dM1: Depends on the previous M mark. Applies
$w^3-(\text{new sum})w^2+(\text{new pair sum})w-(\text{new product})(=0)$ condone the use of any letter here.

A1: At least two of a, b, c, d correct
A1: Fully correct equation in term of w

Question	Scheme	Marks	AOs
3(a)	$(5r-2)^2 = 25r^2 - 20r + 4$	B1	1.1b
	$\sum_{r=1}^{n} 25r^2 - 20r + 4 = \frac{25}{6}n(n+1)(2n+1) - \frac{20}{2}n(n+1) + \ldots$	M1	2.1
	$= \frac{25}{6}n(n+1)(2n+1) - \frac{20}{2}n(n+1) + 4n$	A1	1.1b
	$= \frac{1}{6}n\left[25(2n^2+3n+1) - 60(n+1) + 24\right]$	dM1	1.1b
	$= \frac{1}{6}n\left[50n^2 + 15n - 11\right]$	A1	1.1b
		(5)	
(b)	$\frac{1}{6}k\left[50k^2 + 15k - 11\right] = 94k^2$	M1	1.1b
	$50k^3 - 549k^2 - 11k = 0$ or $50k^2 - 549k - 11 = 0$	A1	1.1b
	$(k-11)(50k+1) = 0 \Rightarrow k = \ldots$	M1	1.1b
	$k = 11$(only)	A1	2.3
		(4)	
			(9 marks)

Notes

(a)

B1: Correct expansion

M1: Substitutes at least one of the standard formulae into their expanded expression

A1: Fully correct expression

dM1: Attempts to factorise $\frac{1}{6}n$ having used at least one standard formula correctly. Dependent on the first M mark.

A1: Obtains the correct expression or the correct values of a, b and c

(b)

M1: Uses their result from part (a) and sets equal to $94k^2$ and attempt to expand and collect terms.

A1: Correct cubic or quadratic

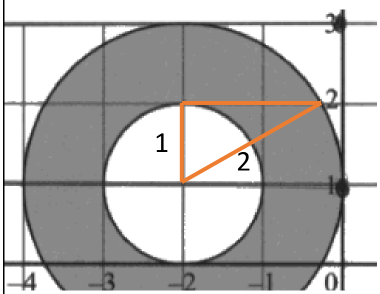
M1: Attempts to solve their 3TQ or cubic equation

A1: Identifies the correct value of k with no other values offered

Question	Scheme	Marks	AOs
4(a)	$\mathbf{MN} = \begin{pmatrix} 2k-24 & 0 & 0 \\ k^2-7k+10 & 6k-44 & -10k+50 \\ 4k-20 & 0 & -14 \end{pmatrix}$	B1 B1	1.1b 1.1b
		(2)	
(b)(i)	$\mathbf{MN} = \begin{pmatrix} -14 & 0 & 0 \\ 0 & -14 & 0 \\ 0 & 0 & -14 \end{pmatrix}$	B1ft	1.1b
(ii)	$\mathbf{M}^{-1} = -\frac{1}{14}\begin{pmatrix} -2 & 6 & -10 \\ 2 & -20 & 24 \\ -3 & 2 & -1 \end{pmatrix}$	B1	1.1b
		(2)	
(c)	$\mathbf{M}^{-1} = -\frac{1}{14}\begin{pmatrix} -2 & 6 & -10 \\ 2 & -20 & 24 \\ -3 & 2 & -1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} = \ldots$	M1	1.1b
	$\left(-\frac{12}{7}, \frac{40}{7}, -\frac{1}{14}\right)$	A1	1.1b
		(2)	
(d)	The coordinates of the only point at which the **planes** represented by the equations in (c) meet.	B1	2.2a
		(1)	
		(7 marks)	

Notes

(a)
B1: For 2 correct rows or 2 correct columns (allow unsimplified)
B1: Fully correct simplified matrix
(b)(i)
B1ft: Correct matrix (follow through from part (a)). If an error with part (a) allow the correct matrix stated, restart use of calculator.
(ii)
B1: Deduces the correct inverse matrix, may use calculator
(c)
M1: Any complete method to find the values of x, y and z (Must be using **their inverse** if using the method in the main scheme)
Allow use of a calculator
A1: Correct exact coordinates (allow as a vector or $x = \ldots, y = \ldots, z = \ldots$)
(d)
B1: Describes the correct geometrical configuration of the **planes**

Question	Scheme	Marks	AOs
5(a)	$a = 1, d = 2$	B1	1.1b
	$b = 2$	B1	1.1b
	$c = -1$	B1	1.1b
		(3)	
(b)	$\|z-\text{i}\| = \|z-3\text{i}\| \Rightarrow y = 2$	B1	2.2a
	Area between the circles $= \pi\times2^2 - \pi\times1^2$	M1	1.1a
	 Angle subtended at centre = $2\times\cos^{-1}\left(\frac{1}{2}\right)$ Alternatively $(x+2)^2+(y-1)^2=4,\ y=2\Rightarrow x=...$ Or $x=\sqrt{2^2-1^2}$ Leading to Angle subtended at centre = $2\times\tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$	M1	3.1a
	Segment area = $\frac{1}{2}\times\frac{2\pi}{3}\times2^2-\frac{1}{2}\times2^2\times\sin\left(\text{"}\frac{2\pi}{3}\text{"}\right)\left\{=\frac{4}{3}\pi-\sqrt{3}\right\}$	M1 A1	2.1 1.1b
	Area of Q: $\pi\times2^2-\pi\times1^2-\left(\frac{1}{2}\times\frac{2\pi}{3}\times2^2-\frac{1}{2}\times2^2\times\sin\left(\frac{2\pi}{3}\right)\right)$	M1	3.1a
	$=\frac{5\pi}{3}+\sqrt{3}$	A1	1.1b
		(7)	
			(10 marks)

Notes

(a)
B1: Correct values for a and d
B1: Correct value for b
B1: Correct value for c
(b)
B1: Deduces that $|z-\text{i}| = |z-3\text{i}|$ is a perpendicular bisector with equation $y = 2$, this may be drawn on a diagram.
M1: Selects the correct procedure to find the area of the large circle – the area of the small circle.
M1: Correct method to find the angle at the centre (or half this angle).
Recognises that the hypotenuse is the radius of the larger circle and the adjacent is the radius if the smaller circle and using cosine
Alternatively find where the perpendicular bisector intersects the larger circle so uses their $y = 2$ and the equation of the larger circle in an attempt to establish the x values for the intersection points or uses geometry and Pythagoras to identify the required length and then uses tangent.
M1: Correct method for the area of the minor segment (allow equivalent work)

A1: Correct expression M1: Fully correct strategy for the required area. Must be subtracting the area of the minor segment from the annulus area. A1: Correct exact answer Note: 6.968

Question	Scheme	Marks	AOs
6(a)	Any two of: $\begin{cases} \pm k\overrightarrow{AB} = \pm k(5\mathbf{i}+25\mathbf{j}+5\mathbf{k}), \\ \pm k\overrightarrow{AC} = \pm k(-15\mathbf{i}+15\mathbf{j}-10\mathbf{k}), \\ \pm k\overrightarrow{BC} = \pm k(-20\mathbf{i}-10\mathbf{j}-15\mathbf{k}) \end{cases}$	M1	3.3
	Let normal vector be $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ $(a\mathbf{i}+b\mathbf{j}+c\mathbf{k})\bullet(\mathbf{i}+5\mathbf{j}+\mathbf{k})=0,\ (a\mathbf{i}+b\mathbf{j}+c\mathbf{k})\bullet(-3\mathbf{i}+3\mathbf{j}-2\mathbf{k})=0$ $\Rightarrow a+5b+c=0,\ -3a+3b-2c=0 \Rightarrow a=...,\ b=...,\ c=...$ Alternative: cross product $\begin{vmatrix} 1 & 5 & 1 \\ -3 & 3 & -2 \end{vmatrix} = (-10-3)\mathbf{i}-(-2+3)\mathbf{j}+(3+15)\mathbf{k}$	M1	1.1b
	$\mathbf{n}=k(-13\mathbf{i}-\mathbf{j}+18\mathbf{k})$	A1	1.1b
	$(-13\mathbf{i}-\mathbf{j}+18\mathbf{k})\bullet(10\mathbf{i}+5\mathbf{j}-50\mathbf{k})=...$	M1	1.1b
	$\mathbf{r}\bullet(13\mathbf{i}+\mathbf{j}-18\mathbf{k})=1035$ o.e. $\mathbf{r}\bullet(-13\mathbf{i}-\mathbf{j}+18\mathbf{k})=-1035$ $\mathbf{r}\bullet(325\mathbf{i}+25\mathbf{j}-450\mathbf{k})=25875$	A1	2.5
		(5)	
(b)	Attempts the scalar product between their normal vector and the vector **k** and uses trigonometry to find an angle	M1	3.1b
	$(-13\mathbf{i}-\mathbf{j}+18\mathbf{k})\bullet\mathbf{k}=-18=\sqrt{13^2+1^2+18^2}\cos\alpha$	M1	1.1b
	$\cos\alpha=\dfrac{-18}{\sqrt{494}} \Rightarrow \alpha=144.08... \Rightarrow \theta=36°$	A1	3.2a
		(3)	
(c)	Distance required is $\lvert\lambda\rvert$ where $\begin{pmatrix}13\\1\\-18\end{pmatrix}\bullet\begin{pmatrix}5\\12\\\lambda\end{pmatrix}=1035$	M1	3.4
	$\lvert\lambda\rvert = 53.2\text{m}$	A1	1.1b
		(2)	
(d)	E.g. • The mineral layer will **not** be perfectly **flat/smooth** and will not form a plane • The mineral layer will have a **depth** and this should be taken into account	B1	3.5b

		(1)	
			(11 marks)

Notes

(a)
M1: Attempts to find at least 2 vectors in the plane that can be used to set up the model. Two correct value implies the correct method if not explicitly seen.
M1: Attempts a normal vector using an appropriate method. E.g. as in main scheme or may use vector product
A1: A correct normal vector
M1: Applies **r.n** = *d* with their normal vector and a point in the plane to find a value for *d*
A1: Correct equation (allow any multiple)
(b)
M1: Realises the scalar product between their from part (a) and a vector parallel to **k** and so applies it and uses trigonometry to find an angle
M1: Forms the scalar product between their from part (a) and a vector parallel to **k**
A1: Correct angle
(c)
M1: Uses the model and a correct strategy to establish the distance from (5, 12, 0) to the plane vertically downwards
A1: Correct distance
(d)
B1: Any reasonable limitation – see scheme

Question	Scheme	Marks	AOs
7(a)(i)	$2 - \mathrm{i}$	B1	1.2
(ii)	Roots of polynomials with real coefficients occur in conjugate pairs, β and γ form a conjugate pair, α is real so δ must also be real. or Quartics have either 4 real roots, 2 real roots and 2 complex roots or 4 complex roots. As 2 complex roots and 1 real root therefore so δ must also be real. or As α real and only one root δ remaining, if complex it would need to have a complex conjugate, which it can't have so must be real	B1	2.4
		(2)	
(b)	$\alpha+\beta+\gamma+\delta=6$ $\Rightarrow 3+2+\mathrm{i}+2-\mathrm{i}+\delta=6 \Rightarrow \delta=...$	M1	3.1a
	$\delta=-1$	A1	1.1b
		(2)	
(c)	$\mathrm{f}(z)=(z-3)(z+1)(z-(2+\mathrm{i}))(z-(2-\mathrm{i}))=...$ Alternative pair sum $=(3)(2+\mathrm{i})+(3)(2-\mathrm{i})+(3)(-1)+(-1)(2+\mathrm{i})$ $+(-1)(2-\mathrm{i})+(2+\mathrm{i})(2-\mathrm{i})=...\{10\}$ triple sum $=(3)(2+\mathrm{i})(2-\mathrm{i})+(3)(-1)(2+\mathrm{i})$ $+(3)(-1)(2-\mathrm{i})+(-1)(2+\mathrm{i})(2-\mathrm{i})=...\{-2\}$ product $=(3)(2+\mathrm{i})(2-\mathrm{i})(-1)=...\{-15\}$	M1	3.1a
	$=(z^2-2z-3)(z^2-4z+5)$ $=z^4-6z^3+10z^2+2z-15$ $p=10, q=2, r=-15$	A1 A1	1.1b 1.1b
		(3)	
(d)	$z=\frac{1}{2}, -\frac{3}{2}$	B1ft	1.1b
	$z=-1\pm\frac{\mathrm{i}}{2}$	B1ft	1.1b
		(2)	
			(9 marks)

Notes

(a)(i)
B1: Correct complex number
(a)(ii)
B1: Correct explanation.
(b)
M1: Uses 2 ± i and 1 together with the sum of roots = ± 6 to find a value for δ
A1: Correct value
(c)

M1: Uses (z – 3) and (z – their δ) and their conjugate pair correctly as factors and makes an attempt to expand Alternatively attempts to find the pair sum, triple sum and product A1: Establishes at least 2 of the required coefficients correctly A1: Correct quartic or correct constants (d) B1ft: For $-\frac{3}{2}$ and $-\frac{\delta}{2}$ as the real roots B1ft: For $-1-\frac{i}{2}$ and $-\frac{\gamma}{2}$ as the complex roots

Question	Scheme	Marks	AOs
8(a)	$n=1,\ \ \text{lhs}=1(2)(3)=6,\quad \text{rhs}=\frac{1}{2}(1)(2)^2(3)=6$ (true for n = 1)	B1	2.2a
	Assume true for $n = k$ so $\sum_{r=1}^{k} r(r+1)(2r+1)=\frac{1}{2}k(k+1)^2(k+2)$	M1	2.4
	$\sum_{r=1}^{k+1} r(r+1)(2r+1)=\frac{1}{2}k(k+1)^2(k+2)+(k+1)(k+2)(2k+3)$	M1	2.1
	$=\frac{1}{2}(k+1)(k+2)\left[k(k+1)+2(2k+3)\right]$	dM1	1.1b
	$=\frac{1}{2}(k+1)(k+2)\left[k^2+5k+6\right]=\frac{1}{2}(k+1)(k+2)(k+2)(k+3)$ Shows that $=\frac{1}{2}(\underline{k+1})(\underline{k+1}+1)^2(\underline{k+1}+2)$ Alternatively shows that $\sum_{r=1}^{k+1} r(r+1)(2r+1)=\frac{1}{2}(k+1)(k+1+1)^2(k+1+2)$ $=\frac{1}{2}(k+1)(k+2)^2(k+3)$ Compares with their summation and concludes true for $n = k$ +1, may be seen in the conclusion.	A1	1.1b
	If the statement is **true for *n* = *k* then** it has been shown **true for *n* = *k* + 1** and as it is **true for *n* = 1**, the statement is **true** for all **positive integers *n*.**	A1	2.4
		(6)	
(b)	$\sum_{r=n}^{2n} r(r+1)(2r+1)=\frac{1}{2}(2n)(2n+1)^2(2n+2)-\frac{1}{2}(n-1)n^2(n+1)$	M1	3.1a
	$=\frac{1}{2}n(n+1)\left[4(2n+1)^2-n(n-1)\right]$	M1	1.1b
	$=\frac{1}{2}n(n+1)(15n^2+17n+4)$ $=\frac{1}{2}n(n+1)(3n+1)(5n+4)$	A1	1.1b
		(3)	
			(9 marks)

Notes
(a) Note ePen B1 M1 M1 A1 A1 A1 B1: Substitutes n = 1 into both sides to show that they are both equal to 6. (There is no need to state true for n = 1 for this mark) M1: Makes a statement that assumes the result is true for some value of n, say k M1: Adds the (k + 1)th term to the assumed result dM1: Dependent on previous M, factorises out $\frac{1}{2}(k+1)(k+2)$ A1: Reaches a correct the required expression no errors and shows that this is the correct sum for $n = k+1$ A1: Depends on all except **B** mark being scored (must have been some attempt to show true for n = 1). Correct conclusion conveying all the points in bold. (b) M1: Realises that $\sum_{r=1}^{2n} r(r+1)(2r+1) - \sum_{r=1}^{n-1} r(r+1)(2r+1)$ is required and uses the result from part (a) to obtain the required sum in terms of n M1: Attempts to factorise by $\frac{1}{2}n(n+1)$ A1: Correct expression or correct values

Question	Scheme	Marks	AOs
9(a)	$(5, 15) \Rightarrow 15 = \frac{\sqrt{225\times 5^2 - 2025}}{a} \Rightarrow a = ...$	M1	3.3
	$a = 4$	A1	1.1b
		(2)	
(b)	Evidence of the use of $\pi\int x^2\, dy$ for the curve BC or the curve CD	M1	3.1b
	For BC $V_1 = \frac{\pi}{225}\int(16y^2 + 2025)\, dy$ or $\pi\int\left(\frac{16}{225}y^2 + 9\right)dy$	A1ft	1.1b
	For CD $V_2 = 25\pi\int(16 - y)\, dy$ or $\pi\int(400 - 25y)\, dy$	A1	1.1b
	$V_1 = \frac{\pi}{225}\int_0^{15}(16y^2 + 2025)\, dy$ or $\pi\int_0^{15}\left(\frac{16}{225}y^2 + 9\right)dy$	M1	3.3
	$V_2 = 25\pi\int_{15}^{16}(16 - y)\, dy$ or $\pi\int_{15}^{16}(400 - 25y)\, dy$	M1	3.3
	$V_1 = \frac{\{\pi\}}{225}\left[\frac{16y^3}{3} + 2025y\right]_0^{15}$ or $\{\pi\}\left[\frac{16y^3}{675} + 9y\right]_0^{15}$	A1ft	1.1b
	$V_2 = 25\{\pi\}\left[16y - \frac{y^2}{2}\right]_{15}^{16}$ or $\{\pi\}\left[400y - \frac{25y^2}{2}\right]_{15}^{16}$	A1ft	1.1b
	$V = V_1 + V_2 = \frac{\pi}{225}(18000 + 30375) + 25\pi\left(128 - \frac{255}{2}\right)$ $V = V_1 + V_2 = 215\pi + 12.5\pi$	M1	3.4
	$V = \frac{455\pi}{2}\text{cm}^3$ or $227.5\pi\,\text{cm}^3$	A1	2.2b
		(9)	

(c)	E.g. • The equation of the curve may not be a suitable model • The sides of the candle will not be perfectly curved/smooth • There will be a whole in the middle for the wick	B1	3.5b
		(1)	
(d)	Makes an appropriate comment that is consistent with their value for the volume and 700 cm^3. E.g. a good estimate as 700 cm^3 is only 15 cm^3 less than 715 cm^3	B1ft	3.5a
		(1)	
			(13 marks)

Notes

(a)

M1: Substitutes (5, 15) into the equation modelling the curve in an attempt to find the value of a

A1: Infers from the data in the model, the value of a

(b)

M1: Uses either model to obtain x^2 in terms of y and applies $\pi \int x^2 \, \mathrm{d}y$

A1ft: Correct expression for the volume generated by the curve BC (follow through their a value)

A1: Correct expression for the volume generated by the curve CD

M1: Chooses limits appropriate to their model for the curve BC

M1: Chooses limits appropriate to their model for the curve CD

A1ft: Correct integration (follow through their a value)

A1ft: Correct integration follow through on their volume as long it is of the form $Ay - By^2$

M1: Uses the model to find the sum of volumes

A1: $\dfrac{455\pi}{2}$

Note: Use of calculator for integration maximum score M1 A1ft A1 M1 M1 A0ft A0ft M1 A1

(c)

B1: States an acceptable limitation of the model

(d)

B1ft: Compares the actual volume to their answer to part (b) and makes an assessment of the model with a reason.